

A starquake model for Vela pulsar

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ABSTRACT

The measured values of glitch healing parameter, Q , of the Vela pulsar are found to be inconsistent with the starquake mechanism of glitch generation in various neutron star (NS) models, based upon the parameterized equations of state (EOSs) of dense nuclear matter. Since such models correspond to an unrealistic mass range $\leq 0.5M_{\odot}$ for the pulsar, if the observational constraints of the fractional moment of inertia of the core component ($I_{\text{core}}/I_{\text{total}} \leq 0.2$) which equals the glitch healing parameter, Q , in the starquake model, are imposed on these models. However, we show that these observational constraints yield a realistic mass range for NS models, corresponding to a core given by the stiffest equation of state (EOS), $dP/dE = 1$ (in geometrized units) and the envelope is characterized by the well known EOS of adiabatic polytrope $d \ln P / d \ln \rho = \Gamma_1$, if the continuity of the *adiabatic speed of sound* ($v = \sqrt{dP/dE}$) together with pressure (P), energy-density (E), and the two metric parameters (ν , and λ) is assured at the core-envelope boundary of the models and this boundary is worked out on the basis of the ‘compatibility criterion’ for hydrostatic equilibrium. The models yield the stable sequence of NS masses in the range, $1.758M_{\odot} \leq M \leq 2.2M_{\odot}$, corresponding to the values of glitch healing parameter range, $0 \leq Q \leq 0.197$, for a choice of the ‘transition density’, $E_b = 1.342 \times 10^{15} \text{ g cm}^{-3}$, at the core-envelope boundary. The maximum stable value of $2.2M_{\odot}$ in this sequence, in fact, corresponds to the lowest possible upper bound on NS masses calculated in the literature, on the basis of modern EOSs for NS matter. The models yield the surface redshift $z_R \simeq 0.6913$ and mass $M \simeq 2.153M_{\odot}$ for the “central” weighted mean value, $Q = 0.12 \pm 0.07$, of the glitch healing parameter of the Vela pulsar. This value of mass can increase slightly upto $M \simeq 2.196M_{\odot}$, whereas the surface redshift can increase upto the value $z_R \simeq 0.7568$ (which represents an ultra-compact object (UCO; $z_R \geq 0.73$)), if the observational constraint of the upper weighted mean value of $Q \simeq 0.19$ is imposed on these models. However, for the lower weighted mean value of $Q \simeq 0.05$, the mass and surface redshift can decrease upto the values of $M \simeq 2.052M_{\odot}$ and $z_R \simeq 0.6066$ respectively. These results set the lower bound on the energy of a gravitationally redshifted radiation in the rather narrow range of 0.291 - 0.302 MeV. The observation of the lower bound on the energy of a γ -ray pulse at about 0.30 MeV from the Vela pulsar in 1984 is in excellent agreement with this result, provided this energy could be interpreted as the energy of a gravitationally redshifted electron-positron annihilation radiation from the star’s surface.

Key words: dense matter - equation of state - stars: neutron - stars: pulsars: individual: Vela

1 INTRODUCTION

The glitch (sudden increase in the rotational velocity of a pulsar) data yield the important information regarding the internal structure of neutron stars (NSs), since it provide the best tool for testing various glitch models which are actually

based upon the internal structure of NSs. At present, there are two well accepted glitch models available in the literature: (i) the starquake (Baym & Pines 1971; Ruderman 1976; Alpar et al 1996) and (ii) the vortex unpinning (Anderson & Itoh 1975; Alpar et al 1993) models. And, both of them, in fact, agreed, in general, upon the same conventional notion that a NS may be considered as a two-component structure, a superfluid interior core which contains most of the NS’s

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mass surrounded by a rigid crust which contains only a few percent of the total mass (here the term ‘crust’ is used for the solid crust plus other interior part of the star (strongly coupled to it) right upto the superfluid core; we shall call this portion as ‘envelope’ of the star). The envelope of conventional NS models is characterized by different equations of state (EOSs) in an appropriate sequence below a ‘fiduciary’ transition density, E_b , and the core is sometimes characterized by the extreme causal EOS, $dP/dE = 1$ beyond E_b , in order to calculate an upper bound on NS masses. Each member of the stable mass-radius sequence of such models may represent a ‘realistic’ case.

At present, the most extensive and accurate glitch data for Crab and Vela pulsars are available in the literature. Crawford & Demiański (2003) have collected the all measured values of glitch healing parameter, Q , for Crab and Vela pulsars, which is defined in starquake glitch model as the fractional moment of inertia, i.e. the ratio of the moment of inertia of the superfluid core, I_{core} , to the moment of inertia of the entire configuration, I_{total} , as

$$Q = \frac{I_{\text{core}}}{I_{\text{total}}}. \quad (1)$$

They have calculated that 21 measured values of Q for Crab glitches yields a weighted mean of $Q = 0.72 \pm 0.05$, and the range of $Q \geq 0.7$ encompasses the observed distribution for the Crab pulsar. In order to test the starquake model for Crab pulsar, they have computed Q (as given by Eq.(1)) values for seven representative EOSs of dense nuclear matter, covering a range of neutron star masses. Their study shows that the much larger values of $Q (\geq 0.7)$ for the Crab pulsar is fulfilled by all, but the six EOSs considered in the study corresponding to a ‘realistic’ neutron star mass range $1.4 \pm 0.2 M_\odot$. On the other hand, a weighted mean value of the 11 measurements for Vela yields a much smaller value of $Q (= 0.12 \pm 0.07)$ and the all estimates for Vela agree with the likely range of $Q \leq 0.2$. Thus, their results are found to be consistent with the starquake model predictions for the Crab pulsar. They have also concluded that the much smaller values of $Q \leq 0.2$ for Vela pulsar are inconsistent with the starquake model predictions, since the implied Vela mass based upon their models corresponds to a value $\leq 0.05 M_\odot$, which is too low as compared to the ‘realistic’ NS mass range.

However, it seems really surprising that if the NS’s internal structure is described by the same conventional NS models (as mentioned above), why different kinds of glitch mechanisms are required for the explanation of a glitch! It is apparent from the study of weighted mean value of the glitch healing parameter, Q , for the Vela pulsar that one would require a two-component model of NS such that more than eighty percent of the total moment of inertia of the star should remain confined in the envelope region in order to have the starquake model explanation of Vela glitches. The present study deals with the construction of such a model which is possible if we impose the conditions: (i) together with other variables (pressure, energy-density, both of the metric parameters ν and λ), the (adiabatic) speed of sound $v (\equiv \sqrt{dP/dE})$ should also remain continuous at the core-envelope boundary of the model governed by an EOS of ‘adiabatic’ polytrope, $d \ln P / d \ln \rho = \Gamma_1$, in the envelope region and the extreme causal EOS, $(dP/dE) = 1$, in the core region respectively. (ii) The models obtained in (i)

should follow the ‘compatibility criterion’ that for every assigned value of the ratio of central pressure to central energy-density ($\sigma \equiv P_0/E_0$), the compactness ratio $u (\equiv M/R$; total mass to radius ratio of the static configuration in geometrized units) of the models should remain less than or equal to the compactness ratio of the corresponding sphere of homogeneous density distribution, in order to assure the condition of hydrostatic equilibrium (Negi & Durgapal 2001; Negi 2004a).

The reason for assigning the above mentioned gamma-law EOS for densities below the fiduciary transition density E_b , is not only because of the fact that matching of speed of sound (together with other variables) at the core-envelope boundary is possible in an analytical manner (Negi & Durgapal 2000), but also because such models (for various assigned values of constant Γ_1) provide the upper bound on NS masses *independent* of the EOS of the envelope, if the condition of ‘compatibility criterion’ is included (Negi 2005). In the present context, however, the value of Γ_1 follows itself from the matching conditions at the core-envelope boundary on the basis of ‘compatibility criterion’ and may be looked upon as an ‘average’ (constant) value of Γ_1 below the density range, E_b , if this range could have been specified by various EOSs like, WFF (Wiringa, Fiks & Fabrocini 1988), FPS ((Lorenz, Ravenhall & Pethick, 1993), NV (Negele & Vautherin 1973), or BPS (Baym, Pethick & Sutherland 1971) in an appropriate sequence, as are frequently used by various authors in the conventional models of NSs (see, e.g. Kalogera & Baym 1996; Friedman & Ipser 1987), since the choice of the fiduciary transition density, $E_b = 1.342 \times 10^{15} \text{ g cm}^{-3}$ (which is higher than $4E_{nm}$, where $E_{nm} = 2.7 \times 10^{14} \text{ g cm}^{-3}$ denotes the nuclear matter saturation density) yields an upper bound on stable NS masses, $M_{\text{max}} = 2.2 M_\odot$, for our model which, in fact, corresponds to the lowest possible upper bound on NS masses, independent of E_b if $E_b \geq 4E_{nm}$, calculated by Kalogera & Baym (1996) on the basis of modern EOSs for NS matter, fitted to experimental nucleon-nucleon scattering data and the properties of light nuclei. This result also shows consistency with the reasoning mentioned in the previous sentence regarding the imposition of the ‘compatibility criterion’ on NS models. The reproduction of the measured values of the glitch healing parameter, Q , for the Vela pulsar on the basis of the present study indicates that this pulsar should be much compact (approximately twice) as compared to that of the Crab (see, e.g. Negi 2005) and suggests a further implication (discussed under section 4), if starquake is considered to be a viable mechanism for glitch generation in all pulsars.

2 METHODOLOGY

The metric for spherically symmetric and static configurations can be written in the following form

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (2)$$

where ν and λ are functions of r alone. The units are assigned in such a manner that the Newtonian gravitation constant (G) and the speed of light in vacuum (c) both become unity. The Oppenheimer-Volkoff (O-V) equations (Oppenheimer & Volkoff 1939), resulting from the Einstein’s field equations for the systems with isotropic pressure P and

energy-density E can be written as

$$P' = -(P + E)[4\pi Pr^3 + m]/r(r - 2m), \quad (3)$$

$$\nu'/2 = -P'/(P + E), \quad (4)$$

$$m'(r) = 4\pi Er^2; \quad (5)$$

where the prime denotes radial derivative and $m(r)$ is the mass contained within the radius r

$$m(r) = \int_0^r 4\pi Er^2 dr.$$

The core ($0 \leq r \leq b$) of the present model can be written in the following form

$$P = (E - E_s) \quad (6)$$

where E_s is the surface density of the configuration characterized by the extreme causal EOS. While the envelope ($b \leq r \leq R$) is given by the EOS

$$P = K\rho^{\Gamma_1} \quad (7)$$

or

$$(E - \rho) = P/(\Gamma_1 - 1).$$

where K is a constant to be worked out by the matching of various variables at the core-envelope boundary and ρ and Γ_1 represent respectively, the rest-mass density and the (constant) adiabatic index (see, e.g., Tooper 1965).

At the core-envelope boundary, $r = b$, the continuity of $P (= P_b)$, $E (= E_b)$, and $r (= r_b)$ require (Negi & Durgapal 2000)

$$K = P_b/(E_b - [P_b/(\Gamma_1 - 1)])^{\Gamma_1} \quad (8)$$

where Γ_1 is given by

$$\Gamma_1 = [(P + E)/P](dP/dE).$$

The continuity of (dP/dE) , at the core-envelope boundary requires

$$\Gamma_1 = 1 + (E_b/P_b). \quad (9)$$

Thus, the continuity of (dP/dE) , together with other variables (P, E, ν , and λ) is ensured at the core-envelope boundary of the static and spherically symmetric configuration.

The coupled Eqs.(3), (4), (5), are solved for the model by considering Eq.(6) in the core and Eq.(7) in the envelope along with the boundary conditions (8) and (9) at the core-envelope boundary, $r = b$, and the boundary conditions, $P = E = 0$, $m(r = R) = M$, $e^\nu = e^{-\lambda} = (1 - 2M/R) = (1 - 2u)$ at $r = R$, at the surface of the configuration, such that for each possible value of σ , the compactness parameter of the whole configuration always turns out to be less than or equal to the compactness parameter of the corresponding sphere (with the same σ) of the homogeneous density distribution. It is seen that this condition is fulfilled if the *minimum* value of the ratio of pressure to energy-density, P_b/E_b , at the core-envelope boundary reaches about 2.92×10^{-1} . The results of the calculations are presented in Table 1 and 2 respectively, while the mass-radius diagram is shown in Fig.1 for a choice¹ of $E_b = 1.342 \times 10^{15} \text{ g cm}^{-3}$ (which is higher than $4E_{nm}$, where $E_{nm} = 2.7 \times 10^{14} \text{ g cm}^{-3}$ is the nuclear matter

saturation density). We find that the *first* maxima in mass is reached to a value of $2.2M_\odot$ for a σ value about 0.6285 which is the evidence that the models become pulsationally stable up to the maximum value of mass $M_{\max} = 2.2M_\odot$. The radius corresponding to this maximum mass is obtained as 9.587 km. Thus, the maximum compactness, u_{\max} , for this stable model yields the value $\simeq 0.3389$, as shown in Table 1. This upper bound is found to be fully consistent with the *exact* absolute upper bound on compactness ratio of NSs compatible with causality and pulsationally stability (see, e.g. Negi 2004b), and provides evidence regarding the appropriateness of the model with the $\Gamma_1 = \text{constant}$ envelope. The binding energy per unit rest-mass $\alpha_r [\equiv (M_r - M)/M_r]$; where M_r is the rest-mass (see, e.g. Zeldovich & Novikov 1978)] also approaches its *first* maxima of about 0.2314 for the *first* maximum value of mass up to which the configurations remain pulsationally stable.

3 AN APPLICATION OF THE MODELS TO VELA PULSAR

For slowly rotating configurations like Vela pulsar (rotation velocity, Ω , about 70 rad sec^{-1}) the moment of inertia may be calculated in the first order approximation that appears in the form of Lense-Thirring frame dragging-effect. In such situations, however, it appears very useful to use an approximate, but very precise empirical formula which is based on the numerical results obtained for thirty theoretical EOSs of dense nuclear matter. For NSs, the formula yields in the following form (Bejger & Haensel 2002)

$$I \simeq \frac{2}{9}(1 + 5x)MR^2, \quad x > 0.1 \quad (10)$$

where x is the compactness parameter measured in units of $[M_\odot(\text{km})/\text{km}]$, i.e.

$$x = \frac{M/R}{M_\odot/\text{km}} = \frac{u}{1.477}. \quad (11)$$

Eq.(10) is used, together with coupled Eqs.(3 - 5), to calculate the fractional moment of inertia given by Eq.(1) and the moment of inertia of the entire configuration for models presented in Table 1 and Fig.1 respectively. The results are shown in Table 2 and Fig.2 respectively. For the central weighted mean value $Q \simeq 0.12$ for the Vela pulsar, Fig.2 yields the mass value $M \simeq 2.153M_\odot$, radius $R \simeq 9.780 \text{ km}$, and surface redshift $z_R \simeq 0.6913$ respectively. The corresponding core mass, M_B , turns out to be about $0.810M_\odot$ as shown in Table 2 which is about 37.6% of the total mass of the entire configuration. This value of Vela mass can exceed upto a value about $2.196M_\odot$ if the upper limit of the central weighted mean value of $Q \simeq 0.19$ is considered. The corresponding values of surface redshift and core mass can exceed upto the values 0.7568 and $1.046M_\odot$ respectively. This value of surface redshift ($z_R \simeq 0.7568$), in fact, represents an ultra-compact object (UCO; $z_R \geq 0.73$) which are entities of interest (see, e.g. Negi & Durgapal 1999; and references therein). For the lower limit of the 'central' weighted mean value of $Q \simeq 0.05$, the Vela mass can reduce upto a value of about $2.052M_\odot$. The corresponding values of surface redshift and core mass turn out to be 0.6066 and $0.489M_\odot$ respectively. Thus, the core mass of the structure varies between 23.8 % to 47.6% of the total mass, for the lower limit of $Q \simeq 0.05$

¹ this choice of transition density and its subsequent implications are discussed under section 4 of the present study.

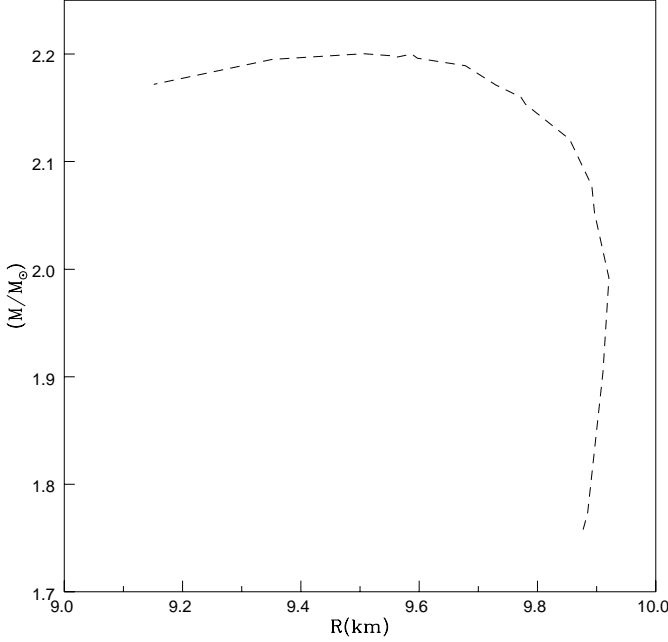


Figure 1. Mass-Radius diagram of the models as discussed in the text for an assigned value of the transition density $E = E_b = 1.342 \times 10^{15} \text{ g cm}^{-3}$ at the core-envelope boundary. The minimum value of the ratio of pressure to energy-density, (P_b/E_b) , at the core envelope boundary is obtained as 2.92×10^{-1} , such that for an assigned value of σ , the inequality $u \leq u_h$ is always satisfied, as shown in Table 1.

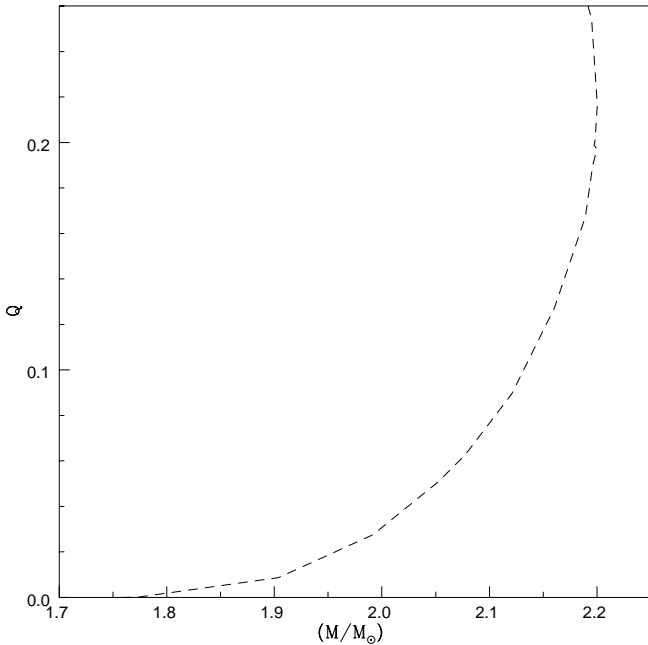


Figure 2. Mass (M/M_\odot) vs. fractional moment of inertia $Q(=I_{\text{core}}/I_{\text{total}})$ for the models as discussed in the text and presented by Table 1 and 2 respectively.

to the upper limit of $Q \simeq 0.19$ respectively. For Q values larger than about 0.197 (i.e., the point of maximum mass $M_{\text{max}} = 2.2M_\odot$), the structures become pulsationally unstable, whereas the minimum stable Vela mass corresponds to a value about $1.758 M_\odot$ as $Q \rightarrow 0$.

4 DISCUSSION

In the previous study, we considered the values of constant $\Gamma_1 = (4/3)$, $(5/3)$, and 2 respectively for the density range below the fiduciary transition density E_b on the basis of ‘compatibility criterion’ in order to construct the star-quake models for Crab pulsar (Negi 2005). If (i) the observational constraint of the glitch healing parameter, and (ii) the observational constraint of the recently evaluated value of the moment of inertia for the Crab pulsar were combined together with the ‘compatibility criterion’ mentioned above, the model with $\Gamma_1 = (5/3)$ envelope itself yielded the value of transitions density, $\simeq 2.7 \times 10^{14} \text{ g cm}^{-3}$, the nuclear matter saturation density at the core-envelope boundary. This value of E_b yields the upper bound on NS masses $M_{\text{max}} \simeq 4.1M_\odot$, independent of the EOS of the envelope (Negi 2005). Not only this value is found fully consistent with the upper bound on NS masses obtained by using modern EOSs for NS matter, fitted to experimental nucleon-nucleon scattering data and the properties of light nuclei, consistent with causality and dynamical stability (see, Fig.2 of Kalogera & Baym 1996, for a fiduciary transition density $E_b = 2.7 \times 10^{14} \text{ g cm}^{-3}$), the upper bound on compactness ratio of these models ($u_{\text{max}} \simeq 0.34$) was also found fully consistent with the ‘absolute’ upper bound on compactness ratio of NSs consistent with causality and pulsational stability (Negi 2004b). The results of this study are also supported by observations, since the existed value of the upper bound on the energy of a gravitationally redshifted γ -ray line at about 0.40 MeV from the Crab pulsar is found consistent with the predicted value in the energy range of about 0.414 - 418 MeV (see, Negi 2005; and references therein).

In the previous study, the value of the ratio of pressure to density at the core-envelope boundary, P_b/E_b , on the basis of ‘compatibility criterion’ was calculated as $\sim 1.065 \times 10^{-2}$, which represents the region very close to the surface of the star (thin crust), and does not require the necessary continuity of the speed of sound (together with other variables) at this boundary which (obviously) corresponds to a region of relatively low density. Since, we were concerned with the larger values of the glitch healing parameter, $Q \geq 0.70$, for the Crab pulsar which demanded that more than 70% of the moment of inertia should remain attached to the core component. However, the much smaller values of the glitch healing parameter for Vela pulsar, $Q \leq 0.20$, require that more than 80% of the moment of inertia should remain confined to the envelope region which obviously require a much larger value of P_b/E_b (thick crust) as compared to the previous case, i.e. a much deeper region of higher density inside the crust where one can naturally assume that the speed of sound approaches the speed of light and is continuous at some ‘fiduciary’ density at the boundary of extreme causal EOS in the core. In the present context of the Vela pulsar, we do not have any information about the Vela’s moment of inertia, as we did have in

the case for the Crab. However, if the Vela and the Crab both are members of the same conventional sequence of the NS models which terminates at the maximum value of NS mass, then we can assign the value of $4.1M_{\odot}$ as an upper bound of NS masses obtained in the previous study, in order to constrain the present models. This assignment gives the Vela's mass about $4.01M_{\odot}$ for the 'central' weighted mean value of the glitch healing parameter $Q \simeq 0.12$ which looks too large, since none of the observational studies claim such higher values of mass for the Vela pulsar. The corresponding value of transition density, E_b , turns out to be about $3.864 \times 10^{14} \text{ g cm}^{-3}$, which also seems unlikely for the normalization of the P_b/E_b ratio of $\sim 2.92 \times 10^{-1}$ calculated for the present models. Notwithstanding, the result of the previous study which is found fully consistent with the present one is the upper bound on NS compactness, $u \simeq 0.34$.

This finding reveals that if starquake is considered to be a viable model for glitch generation in the Vela pulsar² together with the Crab (and, in fact, in all pulsars!) then it would lead to the following implication that there might exist a parallel sequence of the stable NS masses composed of Vela-like pulsars, together with the conventional sequence which composes the Crab-like pulsars, such that this parallel sequence begins with somewhat higher value of the compactness ratio as compared to the conventional sequence but terminates with almost the same value of maximum compactness (e.g., the parallel sequence considered in the present study begins with a value of compactness ratio about 0.2628 which is higher than the corresponding value of compactness ratio of conventional sequence undertaken by Negi (2005), but both of these sequences terminate with almost the same value of maximum compactness, $u \simeq 0.34$) consistent with the absolute upper bound on NS compactness (Negi 2004b). However, such a sequence would correspond to different values of minimum and maximum stable masses as compared to the conventional sequence. Apparently, the minimum mass of this sequence would be higher than the minimum mass of the conventional sequence, whereas the maximum mass of this sequence would correspond to a value less than the maximum mass of the conventional sequence.

Kalogera & Baym (1996) have shown that the lowest possible upper bound on NS masses corresponds to a value of $2.2M_{\odot}$ independent of E_b , provided the fiduciary transition density, E_b , satisfies the condition $E_b \geq 4E_{nm}$. Our models, in fact, satisfy this condition, since for the lowest possible upper bound of $2.2M_{\odot}$, we obtain the value of transition density, $E_b = 1.342 \times 10^{15} \text{ g cm}^{-3}$, which is higher than $4E_{nm}$. Apparently, this value of transition density yields the 'minimum' possible mass for Vela pulsar on the basis of present models as $2.153M_{\odot}$ for the 'central' weighted mean value of $Q \simeq 0.12$. This is discussed earlier under sections 2 and 3 respectively. Thus, in view of the maximum mass and density regime, the results of the present study are fully

consistent with the model based on the modern theory of nuclear interactions and underline the appropriateness of the use of gamma-law EOS in the envelope, the necessity of the continuity of speed of sound across the boundary, and the imposition of the 'compatibility criterion', in order to have a starquake model explanation for Vela-like pulsars.

5 RESULTS AND CONCLUSIONS

The study shows that if starquake is responsible for the glitch generation in Vela pulsar, the surface redshift, z_R , for NS model of Vela pulsar should correspond to a value of 0.6912, if the observational constraint of the 'central' weighted mean value of the glitch healing parameter $Q \simeq 0.12$ is imposed. This value gives the 'minimum' mass of the Vela pulsar $M \simeq 2.153M_{\odot}$ for the 'transition density' $E_b = 1.342 \times 10^{15} \text{ g cm}^{-3}$ at the core-envelope boundary, if the lowest possible upper bound on NS masses is considered to be $2.2M_{\odot}$. The mass is slightly increased upto the value $M \simeq 2.196M_{\odot}$, but the surface redshift can increase upto the value $z_R \simeq 0.7568$ if the observational constraint of the upper weighted mean value of the glitch healing parameter $Q \simeq 0.19$ is imposed. However, for the lower weighted mean value of the glitch healing parameter $Q \simeq 0.05$, the mass and surface redshift can decrease upto $z_R \simeq 0.6066$ and $M \simeq 2.052M_{\odot}$ respectively. For the upper weighted mean value of $Q \simeq 0.19$, the structures represent ultra-compact objects (UCO) which are entities of important astrophysical interest (see, e.g. Negi & Durgapal 1999; and references therein). In the limiting case of $Q \rightarrow 0$, the minimum mass for Vela corresponds to the value of $1.758M_{\odot}$. The confirmation of these results require an observation of the lower bound on the energy of a gravitationally redshifted γ -ray line in the narrow energy range about 0.291 - 0.302 MeV from the Vela pulsar which is in excellent agreement with the observation of the lower bound on the energy of a γ -ray pulse of 0.30 MeV from the Vela pulsar (Tumer et al. 1984), if this energy could be interpreted as the energy of a gravitationally redshifted electron-positron annihilation radiation from the star's surface.

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² The other problems associated with the starquake model for Vela pulsar, like the large size of glitches ($\Delta\Omega/\Omega \sim 10^{-6}$), and the observations of significant amount of change in X-ray flux soon after the glitch occurs (see, e.g. Crawford & Demiański (2003); and references therein) are not discussed in the present paper. However, in view of the higher values of compactness ratio of the present models, the future study in this regard may provide some explanation.

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Table 1. Various values of mass (M/M_\odot), radius $R(\text{km})$, compactness ratio (u), binding-energy per particle (α_r), corresponding to the models discussed in the text for different assigned values of σ . The compactness ratio for homogeneous density distribution is represented by u_h . The slanted values correspond to the limiting case upto which the configuration remains pulsationally stable.

(P_0/E_0)	M/M_\odot	$R(\text{km})$	α_r	u	u_h
0.29202	1.757659	9.877204	0.169796	0.262834	0.262854
0.29800	1.772991	9.884901	0.171393	0.264920	0.265166
0.35452	1.903785	9.910778	0.186960	0.283720	0.284570
0.39993	1.991678	9.920844	0.198449	0.296518	0.297502
0.43923	2.052262	9.896808	0.206909	0.306280	0.307194
0.45699	2.077665	9.892000	0.210533	0.310222	0.311190
0.49397	2.120681	9.854696	0.217014	0.317843	0.318832
0.53331	2.153382	9.780031	0.222426	0.325208	0.326097
0.54099	2.159962	9.772185	0.223498	0.326464	0.327424
0.55999	2.171257	9.729357	0.225572	0.329615	0.330584
0.59146	2.189320	9.678408	0.228886	0.334107	0.335476
0.61902	2.196170	9.597157	0.230666	0.337990	0.339441
<i>0.62850</i>	<i>2.200000</i>	<i>9.587457</i>	<i>0.231448</i>	<i>0.338922</i>	<i>0.340741</i>
0.62936	2.197304	9.563581	0.231138	0.339352	0.340858
0.63148	2.198152	9.560925	0.231311	0.339577	0.341144
0.65148	2.200457	9.506329	0.232207	0.341885	0.343769
0.69998	2.195136	9.351309	0.232781	0.346713	0.349633
0.75294	2.171828	9.151000	0.230592	0.350540	0.355328

Table 2. Various values of mass (M/M_\odot), radius $R(\text{km})$, core mass (M_b/M_\odot), core radius $R_b(\text{km})$, and fractional moment of inertia $I_{\text{core}}/I_{\text{total}}$, corresponding to the models discussed in the text for different assigned values of σ . The corresponding surface and central redshifts are represented by z_R and z_0 respectively. The slanted values correspond to the limiting case upto which the configuration remains pulsationally stable.

(P_0/E_0)	M/M_\odot	$R(\text{km})$	M_b/M_\odot	$R_b(\text{km})$	$I_{\text{core}}/I_{\text{total}}$	z_R	z_0
0.29202	1.757659	9.877204	0.000001	0.084931	0.000000	0.451973	0.961475
0.29800	1.772991	9.884901	0.006151	1.301636	0.000032	0.458402	0.983101
0.35452	1.903785	9.910778	0.171574	3.889230	0.008641	0.520468	1.202564
0.39993	1.991678	9.920844	0.341592	4.844349	0.027604	0.567552	1.394268
0.43923	2.052262	9.896808	0.489306	5.412447	0.050835	0.606561	1.573780
0.45699	2.077665	9.892000	0.554063	5.617970	0.062644	0.623161	1.658789
0.49397	2.120681	9.854696	0.683071	5.969921	0.089515	0.656770	1.846732
0.53331	2.153382	9.780031	0.809999	6.255151	0.120661	0.691314	2.065669
0.54099	2.159962	9.772185	0.833445	6.301950	0.126634	0.697422	2.110140
0.55999	2.171257	9.729357	0.889491	6.406664	0.142236	0.713049	2.225541
0.59146	2.189320	9.678408	0.976141	6.548950	0.167190	0.736086	2.426520
0.61902	2.196170	9.597157	1.045560	6.644820	0.190181	0.756766	2.621490
<i>0.62850</i>	<i>2.200000</i>	<i>9.587457</i>	<i>1.068060</i>	<i>6.672101</i>	<i>0.197133</i>	<i>0.761842</i>	<i>2.688782</i>
0.62936	2.197304	9.563581	1.070062	6.674508	0.198876	0.764196	2.698936
0.63148	2.198152	9.560925	1.074988	6.680161	0.200427	0.765435	2.714632
0.65148	2.200457	9.506329	1.119576	6.727057	0.216372	0.778276	2.871016
0.69998	2.195136	9.351309	1.214118	6.793253	0.254274	0.806058	3.299069
0.75294	2.171828	9.151000	1.294603	6.792263	0.293308	0.829036	3.871991